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Shock Waves in Condensed Media (*).

G. E. Duvall

Shock Dynamics Laboratory, Department of Physics
Washington State University, Pullman, Wash.

In this entire discussion we focus attention on a simple thought experiment. We consider a half-space with free surface normal to the x-axis and located at $x = 0$. The medium of interest lies in the region $x > 0$ or $x < 0$. A uniform pressure is applied to the surface or the surface is given an arbitrary velocity at an arbitrary time, and we inquire about the state of the medium at later times. This apparently restrictive model is in reasonable accord with the geometry and physics of most significant experiments and leads to a great variety of interesting problems.

1. – Basic shock relations.

The continuum differential equations of flow, independent of material properties, are, for one-dimensional plane flow:

$$\partial\varrho/\partial t + \partial\varrho u/\partial x = 0\,, \tag{1}$$

$$\varrho\,\mathrm{d}u/\mathrm{d}t \equiv \varrho\,\partial u/\partial t + \varrho u\,\partial u/\partial x = -\,\partial p/\partial x\,, \tag{2}$$

$$\mathrm{d}E/\mathrm{d}t = -\,p\,\mathrm{d}V/\mathrm{d}t; \qquad V = 1/\varrho\,, \tag{3}$$

where t is time, x is Eulerian space co-ordinate, ϱ is density, u is particle or mass velocity, E is internal energy, and p is compressive stress in the x-direction, including all dynamic forces due to viscosity, stress relaxation, etc.

Application of a pressure to the surface of a half-space produces a region of change propagating out from the surface. If we suppose that a very long time has elapsed since the driving pressure was first applied at the free surface, and that pressure has been held at a constant value, p_1, then the region of change in the resulting flow may be supposed far removed from the driving surface. If the half-space fills the region $x > 0$, we may shift the origin of

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co-ordinates to a point deep within the material and suppose that the region of change connects a uniform undisturbed state at $x = +\infty$ to a uniform compressed state at $x = -\infty$. To implement this model, we seek solutions of eqs. (1)-(3) of the form

$$\varrho = \varrho(\xi)\,, \quad u = u(\xi)\,, \quad p = p(\xi)\,, \quad E = E(\xi)\,, \quad V = V(\xi)\,,$$

where $\xi = x - Dt$ and D is a constant propagation velocity. Let the values of variables in the undisturbed state be designated by subscript « 0 » and those in the uniform state at $x = -\infty$ by subscript « 1 ». Then eqs. (1)-(3) can be integrated to yield the relations

$$\varrho(D - u) = \varrho_0(D - u_0)\,, \tag{4}$$

$$p - p_0 = \varrho_0(D - u_0)^2(V_0 - V) = \varrho_0(D - u_0)(u - u_0)\,, \tag{5}$$

$$E - E_0 = \tfrac{1}{2}(p + p_0)(V_0 - V)\,. \tag{6}$$

Substitution of the final-state variables into eqs. (4)-(6) yields the *jump conditions*. They are particularly useful in the forms:

$$\varrho_0/\varrho_1 = 1 - (u_1 - u_0)/(D - u_0)\,, \tag{7}$$

$$(D - u_0)^2 = V_0^2[(p_1 - p_0)/(V_0 - V_1)]\,, \tag{8}$$

$$E_1 - E_0 = \tfrac{1}{2}(p_1 + p_0)(V_0 - V_1)\,. \tag{9}$$

The undisturbed medium will usually be at rest. In deriving eqs. (4)-(9) it has been assumed to have a velocity u_0. The generality obtained by this assumption will at times be useful.

Any travelling wave which connects end states « 1 » and « 0 » and which satisfies eqs. (7)-(9) is called a *shock wave*. The locus of states (p_1, V_1) which satisfy eqs. (7)-(9) is called the « Rankine-Hugoniot (p, V) curve centered at (p_0, V_0) » or, more simply, the « Hugoniot » or « R-H curve ». It is also sometimes called the « dynamic adiabat » or « shock adiabat ». When the root of eq. (8) is taken, a duality of sign appears. If $D - u_0 > 0$ the compressed state lies to the left, the undisturbed state to the right, and the disturbance is a « forward-facing shock wave ».

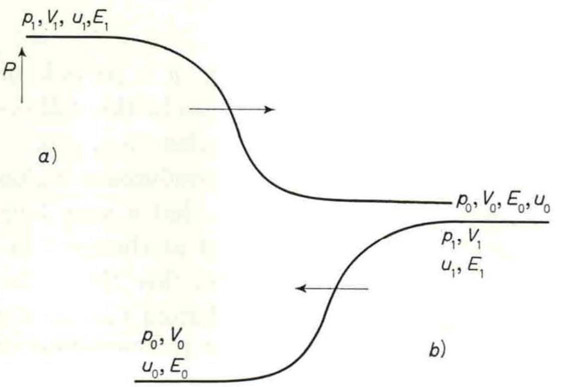

Fig. 1. – *a*) Forward-facing shock wave, $D - u_0 > 0$. *b*) Backward-facing shock wave, $D - u_0 < 0$.